

A NEW THEORY OF CREDIT LINES (WITH EVIDENCE)*

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July 19, 2024

PRELIMINARY

Abstract

We develop a model that suggests a heretofore unexplored role of credit lines: To mitigate debt dilution. The results give a new perspective on the literature on leverage ratchet effects, suggesting they can be curbed by (latent) credit lines. The model explains numerous facts, including why credit lines are pervasive but rarely drawn down and why they are bundled with loans, especially for riskier borrowers. We find that the risk of credit line revocation increases borrower leverage and riskiness, suggesting that limited *bank* commitment can contribute to corporate distress. We find empirical support for this prediction.

*For comments, thanks to Vladimir Asriyan, Nico Crouzet, Peter DeMarzo, Willie Fuchs, Brett Green, Barney Hartman-Glaser, Zhiguo He, Florian Heider, Arvind Krishnamurthy, Uday Rajan, and audiences at the 2024 BIS-CEPR-SCG-SFI Conference on Financial Intermediation, Bonn, the 2024 Kentucky Finance Conference, Mannheim, the 2023 FIFI Conference, the 2024 FIRS conference, the FT Webinar, the 2024 Jackson Hole Conference, UCLA, USC, UT–Dallas, and the 2024 WFA.

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1 Introduction

Credit lines make up the bulk of committed bank credit. Most syndicated loans are bundled with a credit line. But only a small amount of such committed credit is actually utilized, even in times of crisis.¹ Why are credit lines so ubiquitous if so rarely utilized?

We argue that credit lines could play a heretofore overlooked role, even if they are never drawn down: They can mitigate debt dilution. Intuitively, if new lenders anticipate that a borrower will draw a credit line, they are reluctant to lend, knowing their debt will be immediately diluted. Thus, credit lines can serve as a commitment device, deterring new debt and thus protecting existing debt. Indeed, loan-credit line bundles can be designed to implement the commitment/exclusive allocation. The model suggests that lender commitment can substitute for borrower commitment, as they must commit to extend credit lines even when they would make losses. As such, a decrease in lender commitment increases borrowers' temptation to dilute, a prediction that contrasts with existing theories but that is, we show empirically, borne out in the data.

The background environment in our model resembles those in the literature on leverage dynamics (notably, Admati et al. (2017) and DeMarzo and He (2021)). In it, a borrower B issues debt dynamically to lenders. Issuing some debt yields gains from trade because lenders' valuations exceed the borrower's due to, e.g., differences in preferences/beliefs or tax benefits of debt. But, against these benefits, increasing

¹On credit lines making up the bulk of bank credit and on utilization rates, see Berg, Saunders, and Steffen (2020), Greenwald, Krainer, and Paul (2021), Chodorow-Reich et al. (2021), and Sufi (2009); on bundling, see Figure 1 below as well as indirect evidence in Berg, Saunders, and Steffen (2020), Berger, Zhang, and Zhao (2020), and Berlin, Nini, and Yu (2020). In crises, draw downs are concentrated in large firms that do not use bank term loans for balance sheet borrowing; see Berg, Saunders, and Steffen (2020), Chodorow-Reich et al. (2021), Greenwald, Krainer, and Paul (2021) and Ivashina and Scharfstein (2010).

the quantity Q of debt issued has two costs, (i) a direct cost $c(Q)$, capturing costs to B, including not only coupon payments but also, e.g., costs arising from debt-induced agency costs, and (ii) an indirect cost, capturing a decrease in lenders' value, due to, e.g., increased default probabilities or anything else leading the supply curve to be downward sloping. The key friction is that borrowing is non-exclusive: After borrowing from one lender at date t , B can borrow from another at date $t + dt$ —there is only a period dt of exclusivity—and issuing new debt has an externality on old debt. Our innovation relative to the dynamic corporate finance literature is to allow competition not only for loan contracts, but also for credit lines, viz. options to borrow a pre-specified quantity $\tilde{p}d\tilde{Q}$ against a pre-specified face value $d\tilde{Q}$.²

We begin with two benchmarks. In the first, we switch off non-exclusivity, assuming B can commit to issue debt to only one lender. In this case, B issues debt once and never again. A monopolist in the debt market, B understands that issuing more debt will depress its price, and restricts his issuance to keep the price above the marginal cost. The outcome is as in the static trade-off theory (Kraus and Litzenberger (1973)), in which B achieves his target leverage and keeps it there forever. The optimum, from B's point of view, is attained and there is no role for credit lines.

In the second benchmark, we switch off credit lines, so it resembles the dynamic corporate finance models on debt dilution. In this case, once B has debt in place to one lender, he is tempted to take on new debt from another, passing the costs of new issuance on to the debt in place.³ Thus the more debt B has, the more he

²Although we focus on the corporate finance application, the set-up can also apply more generally. As is well known by now, B can be the monopolistic seller of any durable good (debt here) and the lenders its buyers (Coase (1972)). In that context, credit lines correspond to put options: the option to sell a quantity $d\tilde{Q}$ at price \tilde{p} .

³Formally, this result relies on the assumption that B's cost of debt c is concave. That captures that issuing another dollar of debt matters less if B already has a lot of debt outstanding, as is

wants, and the more he takes on. That is Admati et al.’s (2017) leverage ratchet effect.⁴ So, in this case, B cannot commit to keep quantity low and cannot fetch the monopoly price. If the time dt between dates decreases, so B can commit to one lender for only a shorter period, non-exclusivity becomes more severe, leading B to issue more debt. As lenders anticipate issuance going up, the price they are willing to pay drops as they require a higher rate as compensation for, e.g., the increased default risk of a more levered borrower. In the limit as $dt \rightarrow 0$, the price approaches the marginal cost and all benefits of monopoly are eaten up by non-exclusivity, a result that echoes DeMarzo and He (2021) and Coase (1972).⁵

We move on to study the baseline model, with non-exclusivity across periods and competition for loan-credit line bundles at date 0. We first analyze how credit lines in place at date $t > 0$ change the outcome. Then, solving backward, we solve for optimal bundling at date 0.

Our first main result is that a credit line in place—the option to increase (the face value of) outstanding debt by a quantity $d\tilde{Q}$ at rate $1/\tilde{p}$ —can in fact prevent B from taking on new debt—there is what we call a “ratchet anti-ratchet effect.” To see the idea, first observe that, in a reprise of the ratchet effect, higher leverage makes taking more debt (namely drawing the credit line) more attractive. At a certain point, say Q_0 , lenders know that if they lend an additional amount dQ , B will immediately draw on the credit line, increasing his debt by an additional $d\tilde{Q}$, so his debt outstanding will shoot up to $Q_0 + dQ + d\tilde{Q}$. If $d\tilde{Q}$ is large, as credit lines seem to be in practice, that dilutes their debt so much that they are no longer

typical in the literature.

⁴Earlier papers on how non-exclusivity can induce excessive leverage include, e.g., Bizer and DeMarzo (1992) and Petersen and Rajan (1995).

⁵See also papers formalizing and extending the so-called Coase Conjecture, such as Bulow (1982), Gul et al. (1986), and Stokey (1981).

willing to lend dQ , at least not at a price B would accept. The ratchet effect is self-detering: The anticipation of ratcheting up debt in the future (via the credit line) prevents ratcheting up today.

Our second main result endogenizes the loan-credit line bundle that B takes at date 0. We find that, when lenders offer bundles and B chooses one of them, the only outcome that survives is the exclusive outcome, in which B captures the monopoly rent: B borrows from one lender at date 0 and never borrows from anyone else. The reason is that lenders, competing at date 0, offer the loan that is most attractive to B —the monopoly outcome—bundled with a commitment device—the credit line—that allows B to commit to that outcome.

In line with observed low utilization rates, the credit line in the bundle is never drawn, and hence resembles the latent contracts in the literature on non-exclusive competition, e.g., Parlour and Rajan (2001) and Attar et al. (2019a, 2019b). Unlike in that literature, which emphasizes how non-competitive outcomes can arise, we show that the exclusive/monopoly outcome is restored if lenders compete in bundles. The reason is that the bundles include a device that allows B to commit not to borrow from anyone else later on, so just an instant dt of exclusivity is enough to implement exclusivity forever.

For our third main result, we relax the assumption that lenders have full commitment, assuming instead that their credit lines could be revoked with some probability $1 - \alpha$, in line with empirical evidence (Chodorow-Reich and Falato (2022)). We show that if α is high, so credit lines are unlikely to be revoked, the full commitment outcome is still attained—credit lines need not be enforced perfectly to implement the desired outcome perfectly. But if α is lower, the full commitment outcome cannot be attained. The reason is that (latent) credit lines deter lenders only to the extent

that drawing on them depresses the price of debt. That does not happen if the line is revoked. But they are not useless. We show that the chance that they are not revoked always allows B to commit to limit his debt so the price is above marginal cost, by a margin increasing in α .

We test this prediction using syndicated loan data from DealScan. In light of the findings in Chodorow-Reich and Falato (2022), we proxy for increased revocation risk using negative shocks to lender health, which, as constructed, are likely orthogonal to unobserved borrower characteristics (Chodorow-Reich (2014) and Darmouni (2020)). We find support for our theory: When a borrower’s credit lines become more likely to be revoked, it is *more* likely to borrow. That goes in the opposite direction from results in the literature that do not focus on credit line lenders. But those results hold in our data too: When a borrower’s lenders—not just its credit line lenders—are more likely to suffer a liquidity shock, it is *less* likely to borrow (Chodorow-Reich (2014) and Darmouni (2020)). Together, these findings suggest our result is specific to credit lines and, we think, make our overall findings hard to explain with other theories.

We make several contributions to the literature. Relative to papers on dilution/the leverage ratchet effect, we show that with credit lines, the ratchet effect can be turned on its head, effectively used against itself (off equilibrium) to prevent excessive borrowing (on equilibrium). I.e. we show that credit lines serve as a commitment device, restoring the exclusive-monopoly outcome. Relative to papers on non-exclusive competition/latent contracts, we show that an arbitrarily small amount of exclusivity (i.e. for only a time increment dt) effectively restores competition among creditors. Relative to papers on credit lines (e.g., papers in which credit lines serve as liquidity insurance, such as Holmström and Tirole (1998)), we

suggest a new role of credit lines that is unstudied but consistent with a number of facts. Relative to papers on restoring the static monopoly outcome in the problem of selling a durable good over time (Coase (1972)), we show that put options—i.e. options to sell the good, be it debt or something else—can serve as a commitment device not to sell in the future.

Layout. Section 2 contains the model, Section 3 our benchmarks, Section 4 our main results, and Section 5 our empirical analysis. Section 6 is the Conclusion. The Appendix includes tables and figures (Appendix A), a description of our data and empirical variables (Appendix B), and all proofs (Appendix C).

2 Model

There is a borrower B and a continuum of lenders. Everyone has deep pockets and lives forever, discounting the future at rate ρ .

B's flow payoff is as follows:

$$v_t dt = y dt + p_t dQ_t - c(Q_t) dt, \tag{1}$$

where y is the cash flow over $[t, t + dt)$, Q_t is the stock of outstanding debt, dQ_t is the new debt issued over $[t, t + dt)$, p_t is the unit price of debt issued over $[t, t + dt)$, and c is the cost of outstanding debt. We suppose that $c(0) = 0$, $c' > 0$, and $c'' \leq 0$ (cf. supra note 3) and $c' \geq \bar{c}'$ for a strictly positive constant \bar{c}' . The cost c captures not only the expected coupon payments that must be made given an outstanding stock of debt, but also any debt-induced agency costs.

B's lifetime utility from date t onward is

$$V_t = \int_0^\infty e^{-\rho s} v_{t+s} ds. \tag{3}$$

Lenders' flow payoff from holding a unit of debt given stock Q_t is $\gamma(Q_t)dt$, interpreted as the expected coupon payment. We assume that $\gamma' < 0$ and $\gamma(\infty) = 0$. The first assumption captures dilution: The more debt outstanding, the lower is the expected coupon payment. This amounts to a downward-sloping demand curve, which holds true in all the ratchet-effect-type models. The second assumption is that the expected coupon goes to zero as the stock of debt becomes large.⁷

The value of the lenders' stream of coupons on a unit of debt is

$$\Gamma(Q_t) = \int_0^\infty e^{-\rho s} \gamma(Q_{t+s}) ds. \tag{4}$$

Assumption 1. *We impose the following conditions on c and γ :*

1. *Gains from trade:* $\gamma(0) > c'(0)$,
2. *First order approach:* $\gamma(Q)Q - c(Q)$ is concave.

The first assumption ensures that, at date 0, there is scope for trade between B and the lenders; the second allows us to use the first-order approach to solve for the exclusive benchmark in Section 3.1.

⁶By the way, if $v_\tau \equiv y$ —i.e. there is no issuance—then

$$V_t = y \left(\frac{1}{\rho} + dt \right) \xrightarrow{dt \rightarrow 0} \frac{y}{\rho}. \tag{3}$$

⁷There is a discussion about an analogous assumption in the literature on durable goods monopolists; see, e.g., McAfee and Wiseman (2008), on what are referred to as the “gap” and “non-gap” cases.

Interpretation. Rather than letting the cost c capture expected coupon payments $\gamma(Q)Q$, we could also include them directly in the flow payoff, writing $v_t dt = \pi(Q)dt + p_t dQ_t - \gamma(Q)Q$ instead of $v_t dt = y + p_t dQ_t - c(Q_t)dt$. In that specification, $y - c(Q_t) = \pi(Q_t) - \gamma(Q)Q$, where $\pi(Q_t)$ represents the earnings of the firm, assumed to be increasing then decreasing in the stock of debt, as in the classical trade-off theory.⁸

Contracts. At date 0, lenders post bundles of loans and credit lines; at date $t > 0$, lenders post loans. At each date, B takes a bundle/loan from at most one lender.

Solution concept. The solution concept is Markov perfect equilibrium with state variable equal to B's balance sheet, i.e. his debt and credit lines: At each date, the lenders and B act—lenders post contracts and B chooses one and, if he has a credit line in place, chooses whether to draw it (in full)⁹ or not—such that (i) they maximize their future lifetime payoffs given their beliefs, (ii) their beliefs are consistent, and (iii) B's balance sheet is a sufficient statistic for the history with respect to the actions—i.e. the (i) sequential rationality, (ii) equilibrium, and (iii) Markov conditions.

For a given credit line in place, whether it is drawn or not is a binary variable. That allows us to simplify notation by omitting the explicit dependence on the credit line; we denote the value function without the credit line in place (i.e. after it has been drawn) by V and with it (i.e. before it has been drawn by \tilde{V}).

Later, we also introduce what we think is a mild equilibrium refinement that

⁸We can revisit our parametric assumptions in this specification: $c(0) = 0$ is replaced by $\pi(0) = y$, $c'(Q) > 0$ by $\pi'(Q) < (\gamma(Q)Q)'$, $c'' < 0$ by $\pi''(Q) < 2\gamma'(Q) < 0$, and $c' \geq \bar{c}'$ by $-\pi'(Q) > -\gamma'(Q)Q + \bar{c}'$.

⁹It turns out that partial draw downs are not optimal in equilibrium anyway, a fact that follows shortly from the convexity of the value function.

helps us to establish uniqueness (Assumption 3).

3 Benchmarks and Preliminaries

We start with two benchmarks without credit lines, but only loan contracts. The first is the exclusive allocation in which B can commit not to borrow from multiple lenders, as in the classical trade-off theory/static monopolist's problem. The second is the non-exclusive allocation in which B cannot commit, as in the leverage ratchet effect/dynamic monopolist's problem.

3.1 Benchmark: Full Commitment

We first consider the problem in which B can commit to an issuance policy, in particular can commit not to issue debt to other creditors, so non-exclusivity/debt dilution is not a concern.

At date 0, B chooses a debt policy, Q_t for each t , to maximize the expected present value of his payoffs subject to lenders' participation constraints:

$$\max_{(Q_t)_t} \int_0^{\infty} e^{-\rho t} (y + p_t dQ_t - c(Q_t)) dt \quad (5)$$

$$\text{s.t.} \quad p_t \leq \int_0^{\infty} e^{-\rho s} \gamma(Q_{t+s}) ds. \quad (6)$$

To see the solution, suppose that B issued debt at only one date, date τ , and never again. In that case, integrands above are constants, and the problem reduces to marginal revenue equals marginal cost:

$$(p_{\tau} Q_{\tau})' = \frac{c'(Q_{\tau})}{\rho}. \quad (7)$$

This problem is the same for any date τ , so it is optimal to issue once at date 0, as delaying gains from trade is costly, and then keep debt constant, as formalized next:

Proposition 1 (Exclusive benchmark). *With commitment/exclusive competition, B issues debt only at $t = 0$ with quantity Q^e at price p^e , where Q^e solves*

$$\gamma(Q^e) = c'(Q^e) - \gamma'(Q^e)Q^e \quad (8)$$

and

$$p^e = \frac{\gamma(Q^e)}{\rho}. \quad (9)$$

Here the borrower acts as a monopolist, setting marginal revenue equal to (the PV of) marginal cost. Substituting the price p from the lenders' break-even condition, we have:

$$p^e = \frac{c'(Q^e)}{\rho} - \frac{\gamma'(Q^e)Q^e}{\rho}. \quad (10)$$

Since $\gamma' < 0$, it follows that the price is above the marginal cost. The term $\gamma'(Q^e)Q^e$ reflects that B takes into account that issuing one more unit of debt depresses the price for all debt—hence the $\gamma'(Q^e)$ is multiplied by Q^e .

3.2 Benchmark: No Commitment/Non-Exclusivity

We now consider the case in which B can issue debt continuously but cannot commit to his future issuance, a setting that resembles DeMarzo and He (2021). As allocations now need to be time-consistent, we can solve the recursive formulation of the problem, with value function

$$V_t = v_t dt + e^{-\rho dt} V_{t+dt}. \quad (11)$$

Given the (Markov) state variable is the outstanding debt Q_t , we have that

$$V(Q) = \max_{dQ} \left\{ ydt + p(Q + dQ)dQ - c(Q)dt + e^{-\rho dt} V(Q + dQ) \right\}, \quad (12)$$

where B takes the price function p as given. In the limit as $dt \rightarrow 0$, the equation becomes¹⁰

$$\rho V(Q) = \max_q \left\{ p(Q) + V'(Q) \right\} q + y - c(Q), \quad (13)$$

for any smooth policy $qdt = dQ_t$. The objective is linear in the control q , so it must have coefficient zero:

$$p(Q) + V'(Q) = 0, \quad (14)$$

an equation that also appears in DeMarzo and He (2021).¹¹

Proposition 2 (Non-exclusive benchmark). *In the limit as $dt \rightarrow 0$, the value function is*

$$V(Q) = \frac{1}{\rho} (y - c(Q)), \quad (15)$$

the price is

$$p(Q) = \frac{1}{\rho} c'(Q), \quad (16)$$

¹⁰This follows from standard calculations, heuristically as follows. Substitute $V(Q + qdt) = V(Q) + V'(Q)qdt + O(dt^2)$ into equation (12) and multiply through by $(1 + \rho dt) = e^{\rho dt} + O(dt^2)$ to get

$$V(Q) + \rho dt V(Q) = \left\{ y + p(Q + qdt)qdt - c(Q) \right\} dt(1 + \rho dt) + V(Q) + V'(Q)qdt + O(dt^2)$$

or

$$\rho V(Q) = y + p(Q + dQ)qdt - c(Q) + V'(Q)qdt + \{p(Q + qdt)qdt - c(Q)\}\rho dt + O(dt).$$

¹¹There is the flavor of mixed-strategy equilibrium here, as the optimum is determined not by B's direct incentives but by the market's need to make him indifferent over his choice of control.

and the issuance policy is

$$q = \frac{\gamma(Q) - c'(Q)}{-c''(Q)/\rho}. \quad (17)$$

Equation (16) says that the price equals (the present value of) B's marginal cost. I.e. the price is competitive, per the Coase Conjecture on a durable goods monopolists (Coase (1972)). B would like to ration quantities to keep prices above marginal cost, but is always tempted to issue more.

The result also implies that V'' inherits the sign of $-c''$:

Corollary 1 (Convex value). *The value function is convex, with $V'' = -c''/\rho$.*

Before moving on, we make an assumption on c and γ that controls the rate of change of Q_t :

Assumption 2 (Controlled Buybacks). *Let $(Q_t)_t$ be as in Proposition 2. $\gamma(Q_{t+s}) \leq \gamma(Q_t) \exp(k_0 + k_1 s)$ for constants k_0 and $k_1 < \rho$.*

The assumption says that rate of buy backs $-q$ cannot grow exponentially. That ensures that the value of debt goes to zero as the level goes to infinity and thus ensures lenders are not willing to pay much a highly indebted borrower's new debt, even in anticipation of deleveraging. That helps with the proof of Proposition 3.

4 Results

We now study our baseline model, in which B lacks commitment and lenders can offer credit lines together with their loans at date 0. We show that when lenders can bundle credit lines with their loan offers, the commitment allocations of Section 3.1 obtain. We do so in three steps. First, we show that there is a ratchet effect for credit lines: B draws a credit line if and only if his debt is sufficiently high (Section 4.1).

Second, we show that, as a result, credit lines can act as a self-detering mechanism that prevent lenders to make loan offers, which we call the “ratchet-anti-ratchet effect” (Section 4.2). Third, we show that competition on bundles of loans and credit lines implement the commitment allocation (Section 4.3). After that, we extend the model and study how limited committed in the form of potential credit line revocation affects these predictions (Section 4.4).

4.1 The Ratchet Effect for Credit Lines

Below, we show an analog of Admati et al.’s (2017) leverage ratchet effect, which says that drawing a credit line becomes more attractive as debt increases, a consequence of the concavity of c .

Before that, we define a particular debt level, which we label $Q^{\tilde{c}}$, associated with a credit line $(\tilde{p}, d\tilde{Q})$:

Definition 1 (Associated Debt Level). *For a credit line $(\tilde{p}, d\tilde{Q})$, an associated debt level $Q^{\tilde{c}}$ is one such that B is indifferent to drawing $(\tilde{p}, d\tilde{Q})$ with debt $Q^{\tilde{c}}$:*

$$\tilde{p}d\tilde{Q} + V(Q^{\tilde{c}} + d\tilde{Q}) = \tilde{V}(Q^{\tilde{c}}). \quad (18)$$

We now show that B draws a credit line whenever his debt exceeds its associated debt level:

Proposition 3 (Ratchet Effect). *Let $dt \rightarrow 0$. Suppose B has a credit line $(\tilde{p}, d\tilde{Q})$ in place with associated debt level $Q^{\tilde{c}} \leq Q^e$. As long as the price of the credit line is not too low,*

$$\tilde{p} \geq \frac{1}{\rho} \frac{c(Q^{\tilde{c}} + d\tilde{Q}) - c(Q^{\tilde{c}})}{d\tilde{Q}}, \quad (19)$$

B (weakly) prefers to draw the credit line if and only if $Q \geq Q^{\tilde{c}}$.

Intuitively, because c is concave, when B has more debt the cost of issuing at \tilde{p} is relatively low for high Q . Hence B prefers to draw if $Q \geq Q^{\tilde{c}}$, the indifference point, and not to otherwise.

We conclude this section with an assumption on the set of equilibria we study. We think it is mild, as it matters only if B is indifferent between drawing and not.

Assumption 3 (Tie-breaking rule). *Suppose B has a credit line $(\tilde{p}, d\tilde{Q})$ and debt Q in place. If he is indifferent between drawing and not (equation (18) holds), he does not draw if and only if Q solves equation (19) with equality.*

To interpret the Assumption, observe that in light of Proposition 2, the binding equation (19) can be re-written as

$$\tilde{p}d\tilde{Q} + V(Q + d\tilde{Q}) = \frac{y - c(Q)}{\rho}. \quad (20)$$

Thus the assumption says that B breaks indifference toward not drawing only if he is indifferent between drawing the line (the LHS above) and never borrowing again (the RHS). The next result says that (i) for any debt level, there is a credit line for which the condition is satisfied and, given that credit line, (ii) there is no other debt level for which it is.

Lemma 1 (Existence and Uniqueness). *For any debt level Q and size of credit line $d\tilde{Q}$, there is a unique price such that equation (18) holds with equality.*

For any credit line $(\tilde{p}, d\tilde{Q})$, there is at most one debt level for which it does.

4.2 The Ratchet-Anti-Ratchet Effect

Next, we use the result in Proposition 3 to show that for any debt level $Q^{\tilde{c}} \leq Q^e$, there is an associated credit line $(\tilde{p}, d\tilde{Q})$ with $d\tilde{Q}$ large that makes lenders unwilling

to post a loan contract that B is willing to accept. As a result, the debt level $Q^{\bar{c}}$ is an absorbing state for the process Q_t . The result follows from the ratchet effect (Proposition 3)—if B takes on any debt he also takes on $d\tilde{Q}$ —and the assumption $\gamma(\infty) = 0$ —that lenders' value drops below B's in anticipation of his debt going up.

To see why, suppose first that B with credit line credit line with associated debt level $Q^{\bar{c}}$ (Definition 1) and \tilde{p} above the lower bound in Proposition 3. So at $Q^{\bar{c}}$ B will choose to draw the line if he takes on any new debt $dQ > 0$. Now B prefers *not* to take on the new debt at price p if his payoff from taking the loan and drawing the credit line is less than his payoff from doing neither:

$$pdQ + \tilde{p}d\tilde{Q} + V(Q^{\bar{c}} + dQ + d\tilde{Q}) \leq \tilde{V}(Q^{\bar{c}}). \quad (21)$$

Given that B is indifferent to drawing the line given debt in place $Q^{\bar{c}}$, we can replace the RHS with his payoff from taking $(\tilde{p}, d\tilde{Q})$ given only $Q^{\bar{c}}$ in place to write condition (21), for B not to borrow, as

$$pdQ + \tilde{p}d\tilde{Q} + V(Q^{\bar{c}} + dQ + d\tilde{Q}) \leq \tilde{p}d\tilde{Q} + V(Q^{\bar{c}} + d\tilde{Q}). \quad (22)$$

Rearranging gives an upper bound on the price p of new debt:

$$p \leq \frac{V(Q^{\bar{c}} + dQ + d\tilde{Q}) - V(Q^{\bar{c}} + d\tilde{Q})}{dQ}. \quad (23)$$

The inequality must hold for all dQ . Thus, given V convex (Corollary 1), for any p , it is necessary and sufficient that it holds as $dQ \rightarrow 0$. That limiting condition is:

$$p \leq -V'(Q^{\bar{c}} + d\tilde{Q}) = c'(Q^{\bar{c}} + d\tilde{Q}), \quad (24)$$

having used the expression for V in Proposition 2. Condition (24) says that B does not want to issue at a price below marginal cost. The twist is that the marginal cost is conditional on having drawn the line. Given B will draw, immediately increasing his debt by $d\tilde{Q}$, it is the marginal cost at $Q^{\tilde{c}} + d\tilde{Q}$, not at just $Q^{\tilde{c}}$, that matters.

We now turn to whether lenders are willing to lend. A lender that anticipates B will draw the credit line is willing to offer a loan (p, dQ) if and only if its value exceeds the price, or

$$\Gamma(Q^{\tilde{c}} + dQ + d\tilde{Q}) \geq p. \quad (25)$$

Comparing inequalities (24) and (25), we have that for $d\tilde{Q}$ large enough, lenders will not be able to make offers that B is willing to accept, a fact we use in the following result.¹²

Proposition 4 (Ratchet-Anti-ratchet). *For any debt level $Q^{\tilde{c}} \leq Q^e$, there exists a credit line $(\tilde{p}, d\tilde{Q})$ s.t. inequalities (18) and (19) are satisfied and $d\tilde{Q}$ is large enough so that*

$$\Gamma(Q^{\tilde{c}} + d\tilde{Q}) \leq c'(Q^{\tilde{c}} + d\tilde{Q}). \quad (26)$$

With such credit line in place, the value function is as follows:

$$\tilde{V}(Q) = \begin{cases} \frac{1}{\rho} \left(\gamma(Q^{\tilde{c}})(Q^{\tilde{c}} - Q) + y - c(Q^{\tilde{c}}) \right) & \text{if } Q \leq Q^{\tilde{c}}, \\ \tilde{p}d\tilde{Q} + V(Q + d\tilde{Q}) & \text{otherwise,} \end{cases} \quad (27)$$

with V as in Proposition 2.

The main take-away from Proposition 4 is that any $Q^{\tilde{c}} \leq Q^e$ can be supported by a sufficiently large credit line. Moreover, if B's debt is below $Q^{\tilde{c}}$, then he issues

¹²The statement follows from the fact that as $Q \rightarrow \infty$, $c'(Q) \rightarrow \bar{c}' > 0$, whereas $\Gamma(Q) \rightarrow 0$.

debt to jump to the absorbing state $Q^{\tilde{c}}$; when it is above $Q^{\tilde{c}}$, B draws the credit line. At $Q = Q^{\tilde{c}}$, B is indifferent between drawing or not, which given the tie-breaking rule, implies B will not draw (Assumption 3).

There are three key steps to the proof, which is in Appendix C.6. The first is to show that with a credit line with the properties in the Proposition, $Q^{\tilde{c}}$ is an absorbing state, in line with the discussion above. The second is a characterization of the value function given such a credit line is in place. The last is to show existence.

4.3 Bundling

We now suppose that lenders can offer bundles including loans and credit lines at date 0 and B can take exactly one bundle. After that lenders compete in loans and the equilibrium is as in Proposition 2.

In this case, the outcome is the exclusive contracting outcome of Proposition 1:

Proposition 5 (Credit Line Bundles). *If lenders compete in bundles, there exists an equilibrium in which B chooses a bundle with a loan and a credit line at date 0 and never borrows again nor draws the credit line.*

The loan coincides with exclusive outcome (p^e, Q^e) in Proposition 1.

The credit line $(\tilde{p}, d\tilde{Q})$ is such that B is indifferent to drawing given Q^e (equation (18) holds with $Q^{\tilde{c}} = Q^e$) and $d\tilde{Q}$ ensures that inequality (25) is violated for all $dQ > 0$.

Proposition 5 builds on Proposition 4, which says that a credit line is a commitment device not to borrow in the future. At date 0, B can put a credit line in place and thereby commit not to borrow from anyone else. Thus if lenders can bundle loans with credit lines, an instant of exclusivity—from date 0 to date dt —is just as good

as exclusivity forever. As a result, competition in bundles at date 0 can achieve the same outcome as exclusive competition forever.

As mentioned in the Introduction, this result contrasts with the literature on latent contracts (notably Parlour and Rajan (2001) and Attar et al. (2019a, 2019b)). Although our credit lines, never being drawn, resemble the latent contracts in that literature, the outcomes here do not resemble those there. With (i) exclusivity within periods, albeit arbitrarily short ones, and (ii) competition in bundles, not just loans, the non-competitive outcomes of that literature do not arise.

There is also a practical difference between credit lines in our model and latent (loan) contracts in the literature, namely that whereas a credit line is a contract agreed to between a borrower and a (potential) lender that must be honored, the latent contract is just an offer from a lender that can be retracted. That matters, because, in our model, lenders would prefer not to honor credit lines. At the time that they would be drawn, B is so levered that the rate $1/\tilde{p}$ is too low for the lender to break even. So, whereas credit lines, which are, per the contractual agreement, always available, can support a variety of outcomes, the analogous latent contracts, which can, and will, be retracted when B chooses to take them up, cannot.

4.4 Credit Line Revocation

So far, we have assumed that lenders fully commit to credit lines; they are never revoked. Here we relax that assumption. We assume that, conditional on being drawn, a lender honors the credit line with probability α and defaults, lending nothing, with complementary probability. One motivation for this is that, as discussed in Section 4.3, the credit lines in our model are, by construction, loss making when drawn, so lenders would like to revoke them if they could; another is that the offer-

ing lenders could be distressed themselves and unable to honor their commitments (cf. Chodorow-Reich and Falato (2022)). Either way, as above, B draws the credit line $(\tilde{p}, d\tilde{Q})$ if and only if his debt is (strictly) above the threshold $Q^{\tilde{c}}$, defined by the same equation as above (equation (18)). If B is indifferent between drawing and not, he is also indifferent between drawing with probability α and not.¹³

Likewise, the condition for B not to want to borrow (in which case he draws on $(\tilde{p}, d\tilde{Q})$) is analogous to inequality (21), except with drawing the line replaced with drawing it with probability α :

$$\begin{aligned} ydt + pd\tilde{Q} + \alpha \left\{ \tilde{p}d\tilde{Q} - c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}} + dQ + d\tilde{Q}) \right\} + \\ + (1 - \alpha) \left\{ -c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}} + dQ) \right\} \leq \\ \leq ydt - c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}}), \end{aligned} \quad (29)$$

or, using B's indifference condition (28) to rewrite the RHS and rearranging,

$$\begin{aligned} pe^{\rho dt} \leq -\alpha \frac{V(Q^{\tilde{c}} + dQ + d\tilde{Q}) - V(Q^{\tilde{c}} + d\tilde{Q})}{dQ} + \\ - (1 - \alpha) \frac{V(Q^{\tilde{c}} + dQ) - V(Q^{\tilde{c}})}{dQ}. \end{aligned} \quad (30)$$

Per the argument in Section 4.2 (cf. equation (23)), it is necessary and sufficient that the inequality hold for $dQ \rightarrow 0$, or that

$$p \leq -\alpha V'(Q^{\tilde{c}} + d\tilde{Q}) - (1 - \alpha)V'(Q^{\tilde{c}}). \quad (31)$$

¹³Formally, the analog of the indifference condition (18) in which credit lines are revoked with probability α is:

$$\begin{aligned} ydt + \alpha \left\{ \tilde{p}d\tilde{Q} - c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}} + d\tilde{Q}) \right\} + \\ + (1 - \alpha) \left\{ -c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}}) \right\} = ydt - c(Q^{\tilde{c}})dt + e^{-\rho dt}V(Q^{\tilde{c}}). \end{aligned} \quad (28)$$

The α 's cancel recovering the same equation as in the baseline model.

From here, we have the next result:

Lemma 2 (No New Debt with Revocation). *Suppose B has debt $Q^{\tilde{c}}$ and a revocable credit line $(\tilde{p}, d\tilde{Q})$ in place, such that B is indifferent to drawing the line at $Q^{\tilde{c}}$.*

For $dt \rightarrow 0$, B prefers new debt (p, q) for some q to no loan if and only if

$$p \leq \frac{1}{\rho} \left(\alpha c'(Q^{\tilde{c}} + d\tilde{Q}) + (1 - \alpha) c'(Q^{\tilde{c}}) \right). \quad (32)$$

The result says that B does not want to borrow at a price below marginal cost. The twist is that the marginal cost is conditional on drawing the line successfully with probability α .

We now turn to whether lenders are willing to lend. By the definition of Γ , a lender that anticipates that B will draw the credit line successfully with probability α is willing to offer a loan (p, q) if and only if

$$\alpha \Gamma(Q^{\tilde{c}} + dQ + d\tilde{Q}) + (1 - \alpha) \Gamma(Q^{\tilde{c}} + dQ) \geq p. \quad (33)$$

Just comparing inequalities (32) and (33) gives the next result:

Proposition 6 (Revocation). *Consider the setting of Proposition 4. B does not take on new debt at any price lenders will lend at if and only if*

$$\alpha \Gamma(Q^{\tilde{c}} + d\tilde{Q}) + (1 - \alpha) \Gamma(Q^{\tilde{c}}) \leq \frac{1}{\rho} \left(\alpha c'(Q^{\tilde{c}} + d\tilde{Q}) + (1 - \alpha) c'(Q^{\tilde{c}}) \right). \quad (34)$$

Re-writing (34) gives an expression for the extent to which the price $p = \Gamma(Q^{\tilde{c}})$ can exceed the marginal cost $c'(Q^{\tilde{c}})$:

$$p - \frac{c'(Q^{\tilde{c}})}{\rho} \leq \frac{\alpha}{1 - \alpha} \left(\frac{c'(Q^{\tilde{c}} + d\tilde{Q})}{\rho} - \Gamma(Q^{\tilde{c}} + d\tilde{Q}) \right) \xrightarrow{d\tilde{Q} \rightarrow \infty} \frac{\alpha}{1 - \alpha} \frac{\bar{c}'}{\rho}, \quad (35)$$

having used the assumptions that $c'(\infty) = \bar{c}'$ and $\gamma(\infty) = 0$. That says that for α large, high prices can be sustained; as we showed, the full commitment outcome is attained for $\alpha = 1$ (Proposition 5). But for α smaller, lower prices can be sustained with only the non-exclusive outcome $p = c'(Q^{\bar{c}})$ of Proposition 2 available as $\alpha \rightarrow 0^+$.

5 Empirical Analysis

Here we test the prediction in Proposition 6, that borrowers increase debt when the risk that credit lines are revoked goes up. In light of the findings in Chodorow-Reich and Falato (2022), we use a negative shock to lender health to proxy for an increase in revocation risk. For each borrower i , we construct an overall lender health shock following Chodorow-Reich (2014) and Darmouni (2020) and, analogously, a credit line lender health shock, which captures only the shocks to lenders with credit lines outstanding to borrower i (see Appendix B).

Using syndicated loan data from DealScan, we regress an indicator for a borrower taking on new debt against these lender shocks (as well as controls variables¹⁴):

$$\text{new debt}_i = \alpha + \beta \text{shock}_i + \gamma \text{shock CL}_i + \delta X_i + \varepsilon_i. \quad (36)$$

The findings are in Table 1. In short, we find that $\beta < 0$: In line with the literature, a negative shock to a borrower's lenders leads it to borrow less, presumably because credit supply contracts. However, we find that $\gamma > 0$: In line with our theory, a negative shock to a borrower's credit line lenders leads it to borrow more, possibly because the credit line no longer serves as a commitment device not

¹⁴Controls include the number of syndicates that borrower i had taken debt from before the lender shocks, capturing borrowers' access to/need for credit as well as an indicator for having taken a credit line before the lender shocks, capturing general differences between borrowers that do/do not use credit lines.

to borrow.

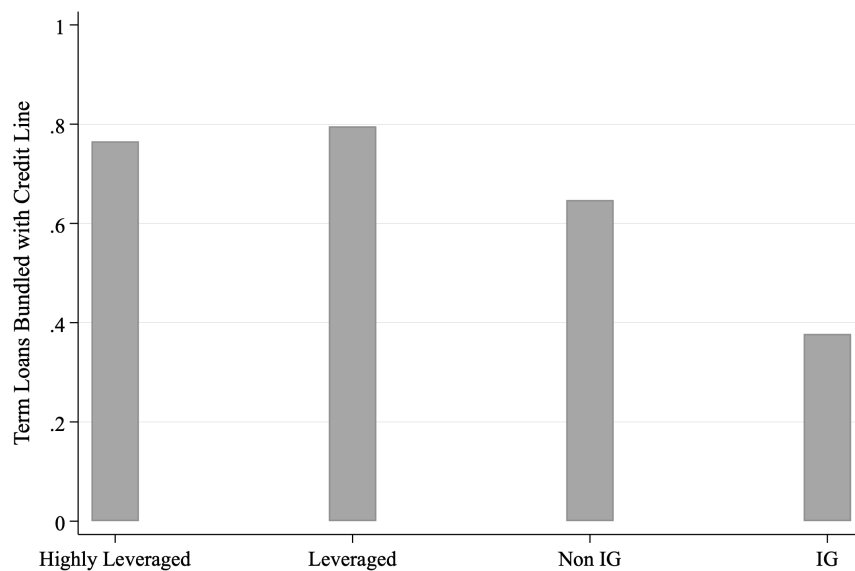
6 Conclusion

We study a model that suggests that credit lines play a heretofore overlooked role. They can mitigate debt dilution. The theory suggests that the option to borrow—viz. a credit line—is valuable even if it is never exercised, explaining why credit lines are ubiquitous but rarely drawn. It also underscores how and why credit lines should be bundled with loans, a pervasive practice never previously studied in the theory literature.

Our paper contrasts with recent corporate finance papers on the leverage ratchet effect, suggesting that including credit lines in the contracting space makes the ratchet effect self-detering, undoing its negative effects. It also contrasts with the literature on latent contracts, suggesting that the kinds of outcomes stressed in that literature might not obtain in dynamic environments.

A Tables and Figures

Figure 1: Bundling Propensity and Firm Riskiness



This figure shows how the proportion of term loans that are bundled with a credit line vary by firm riskiness, as measured by Dealscan’s classification of firms’ market segments. The loans to the safest borrowers are Investment Grade (“IG”), then “Non-IG”, “Leveraged”, and “Highly Leveraged” respectively, where the distinction between the latter three categories depends on pricing thresholds. Data are from Dealscan, covering US C&I syndicated loans from 1997 through 2021 for which at least one lender is a US bank, and excluding financials.

	(1)	(2)	(3)	(4)
Shock	-0.16*** (0.05)	-0.17*** (0.05)	-0.17*** (0.05)	-0.18*** (0.05)
Shock CL		0.03*** (0.01)	0.02** (0.01)	0.03*** (0.01)
Number of Syndicates			0.03*** (0.01)	0.04*** (0.01)
Pre CL Indic				-0.03** (0.01)
Constant	0.20*** (0.04)	0.19*** (0.04)	0.19*** (0.04)	0.18*** (0.04)
Observations	4883	4883	4883	4883
Adjusted R^2	0.002	0.003	0.009	0.010

Table 1: The table reports the effect of credit line revocation risk on firm borrowing per the regression in equation (36). The outcome variable is an indicator for borrowing in the syndicated loan market in the crisis period. The construction of the shocks as well as definition of crisis and normal periods are described in Appendix B. The controls include the number of syndicates firm i borrowed from during normal period and an indicator variable tracking if firm borrowed CL in normal period. Observations are at the firm level. Robust standard errors are reported in parentheses. Two and three stars indicate statistical significance at the 5% and 1% level, respectively.

B Data and Variable Construction

Here we describe details omitted from Section 5.

B.1 Data

We start with the universe of US C&I syndicated loans in DealScan from 1997 through 2021. We classify US C&I loans to be loans that are originated in the US and for which the deal purpose is listed as “general purpose” or “working capital”. We exclude loans to financials.

B.2 Variable Construction

Lender share. In the Dealscan data, the lenders’ share of the loan commitment within a given syndicate are sometimes unreported. In these cases, we impute lenders’ shares following Chodorow-Reich (2014) and Darmouni (2020). Specifically, we calculate the average lender share of lead arrangers and participants separately for each syndicate structure during the time period surrounding the Global Financial Crisis, from 2004 through 2010, among syndicates that do not have missing lender shares.¹⁵ We then fill in the missing lender shares with the average lender shares calculated for the corresponding syndicate structure.

Lender health shocks. For each borrower, we construct overall lender health shocks following Chodorow-Reich (2014) and Darmouni (2020) and, analogously, for lenders with credit lines outstanding to the borrower. Specifically, we define $\Delta L_{b,-i}$ as the decrease in a bank b ’s lending to firms $j \neq i$ in the crisis period vis-à-vis

¹⁵By syndicate structure we mean the number of lead arrangers and the number of other participants in a syndicate.

normal times:

$$\Delta L_{b,-i} := 1 - \frac{2 \sum_{j \neq i} L_{b,j,\text{crisis}}}{\sum_{j \neq i} L_{b,j,\text{normal}}}, \quad (37)$$

where $L_{b,j}$ is the effective number of loan facilities from b to j during normal and crisis times, defined as 10/2005–6/2007 and 10/2008–6/2009, respectively. The effective number of loan facilities is the number of loan facilities originated, with each weighted by the corresponding lender share, as discussed above. We exclude refinancings and amendments (except extensions) during crisis times. We restrict the sample to firms that borrowed during the normal period and banks that are present in both the normal and crisis period. We winsorize $\Delta L_{b,-i}$ at 2%.

We then construct the shocks for each borrower i 's lenders and credit line lenders as weighted sums of $\Delta L_{b,-i}$ over lenders in a borrower i 's last pre-crisis syndicate:

$$\text{Shock}_i = \sum_{b \in S} \alpha_b \Delta L_{b,-i} \quad \text{Shock CL}_i = \sum_{b \in S} \alpha_b^{\text{CL}} \Delta L_{b,-i}, \quad (38)$$

where S is the set of lenders in borrower i 's last pre-crisis syndicate. For a lender $b \in S$, α_b is its average share across all loan facilities in the syndicate and α_b^{CL} is its share within the credit line facility. If borrower i 's last pre-crisis syndicate has no CL or a CL that matures prior to 2008, we set $\text{Shock CL}_i = 0$.

C Proofs

C.1 Proof of Proposition 1

Here we apply a guess-and-verify approach: We assume that it is optimal to set $dQ_t = 0$ for all $t > 0$, solve for the optimal Q_0 , and then show the B cannot benefit by issuing again. Recall that Assumption 1 ensures that our commitment problem has unique solution and thus we can use the first-order approach.

Step 1: Optimal issuance at date 0. The optimal date-0 issuance as if B never issues debt again, $Q_0 = dQ_0$, solves

$$p_0 + \frac{dp_0}{dQ_0}Q_0 - \int_0^\infty e^{-\rho t} c'(Q_0) dt = 0. \quad (39)$$

Using lenders' participation constraint and computing the integrals (which is easy for $Q_t \equiv Q_0$), we can rearrange to find an expression for the optimal Q_0 :

$$p_0 = \frac{\gamma(Q_0)}{\rho} = \frac{c'(Q_0) - \gamma'(Q_0)Q_0}{\rho}. \quad (40)$$

Step 2: No issuance at date $\tau > 0$. Now we verify that the marginal benefit from having $dQ_\tau \neq 0$ for some $\tau > 0$ is zero. To do so, we differentiate the objective

$$p_0 Q_0 + \int_0^\tau e^{-\rho t} (y - c(Q_0)) dt + p_\tau dQ_\tau + \int_\tau^\infty e^{-\rho t} (y - c(Q_\tau)) dt = \quad (41)$$

$$= p_0 dQ_0 + \frac{(y - c(Q_0))}{\rho} (1 - e^{-\rho\tau}) + e^{-\rho\tau} p_\tau dQ_\tau + e^{-\rho\tau} \frac{(y - c(Q_\tau))}{\rho} \quad (42)$$

where $Q_\tau = Q_0 + dQ_\tau$. The FOC is

$$\frac{dp_0}{dQ_\tau}Q_0 + e^{-\rho\tau}p_\tau + e^{-\rho\tau}\frac{dp_\tau}{dQ_\tau}dQ_\tau - \frac{e^{-\rho\tau}}{\rho}c'(Q_0 + dQ_\tau) = 0. \quad (43)$$

Now observe that $dp_0/dQ_\tau = dp_\tau/dQ_\tau$, because

$$p_0 = \int_0^\tau e^{-\rho t}\gamma(Q_0)dt + \int_\tau^\infty e^{-\rho t}\gamma(Q_\tau)dt = \int_0^\tau e^{-\rho t}\gamma(Q_0)dt + e^{-\rho\tau}p_\tau \quad (44)$$

So, substituting from above and cancelling the $e^{-\rho\tau}$, the FOC reads

$$\frac{dp_\tau}{dQ_\tau}dQ_0 + p_\tau + \frac{dp_\tau}{dQ_\tau}dQ_\tau - c'(Q_0 + dQ_\tau) = 0. \quad (45)$$

Using $Q_\tau = Q_0 + dQ_\tau$, we have

$$p_\tau + \frac{dp_\tau}{dQ_\tau} = (p_\tau Q_\tau)' = \frac{c'(Q_0 + dQ_\tau)}{\rho}. \quad (46)$$

I.e. the equation for the static optimum (40). □

C.2 Proof of Proposition 2

The expression for V in (15) follows from substituting the equation from the optimal control (equation (14)) into the continuous-time HJB (equation (13)). The equation for p in (16), comes from differentiating the equation for V (equation (15)) and replacing V' with $-p$ from equation (14).

The issuance policy follows from the law of motion for the price,¹⁶

$$p(Q) = \gamma(Q) + p'(Q)q. \quad (47)$$

Using $p = c'/\rho$, differentiating and rearranging, gives the expression for q in the proposition. \square

C.3 Proof of Corollary 1

The result follows from Proposition 2 and the assumption that $c'' < 0$. \square

C.4 Proof of Proposition 3

We show the result via the following lemmata.

- In Lemma C.1, we rule out smooth equilibria in which B does not draw. That allows us to consider only drawing or jumping.
- In Lemma C.2, we rule out equilibria in which B does not draw today but draws later on. That allows us to consider either drawing immediately or never.

At that point, we have the corollary that, given our focus on Markov equilibria, in any candidate equilibrium in which B does not draw, there must be an absorbing state.

- In Lemma C.3, we show that for Q high, B does not jump to any absorbing state $Q^+ > Q^{\tilde{c}}$.

¹⁶That equation, which can be seen as the Black–Scholes differential equation for a derivative with price p written on an underlying Q following $dQ_t = q_t dt$, is the limit of the standard discounting formula $p(Q) = \gamma(Q)dt + e^{-\rho dt}p(Q + dQ)$.

- In Lemma C.4, we show, symmetrically, that for Q high, B does not jump to any absorbing date $Q^- < Q^{\tilde{c}}$ under a condition on prices, which we show holds in Lemma C.5 and Lemma C.6.
- Finally, in Lemma C.7, we show that B does not draw for $Q < Q^{\tilde{c}}$.

Lemma C.1. *Suppose $\{\tilde{p}, d\tilde{Q}\}$ satisfies (18). Suppose also that the policy is smooth and the value function \tilde{V} is differentiable. B strictly prefers to draw for $Q > Q^{\tilde{c}}$.*

Proof. Suppose (in anticipation of a contradiction) that B prefers not to draw at Q . By continuity, there is a neighborhood of Q for which he strictly prefers not to draw. By smoothness, near Q , \tilde{V} satisfies the HJB as if there were no credit line near Q (i.e. $\rho V(Q) = y + p(Q)q - c(Q) + V'(Q)q$). That equation has the solution $\rho\tilde{V}(Q) = y - c(Q)$. Given c is concave, from (18) we get, $\tilde{V}(Q) \leq \tilde{p}d\tilde{Q} + V(Q + d\tilde{Q})$. That is the desired contradiction. \square

Lemma C.2. *Suppose $\{\tilde{p}, d\tilde{Q}\}$ satisfies (18). If B prefers to draw at t^+ , he prefers to draw at $t < t^+$.*

Proof. Suppose (in anticipation of a contradiction) that B does not draw at t but does at $t^+ = t + dt$. Keeping in mind that, by Lemma C.1, it suffices to focus on jumps and denoting Q_t by Q and Q_{t^+} by Q^+ , that says:

$$\tilde{p}d\tilde{Q} + V(Q + d\tilde{Q}) \leq \tilde{V}(Q) = p(Q^+ - Q) + \tilde{p}d\tilde{Q} + V(Q^+ + d\tilde{Q}), \quad (48)$$

where p is the issuance price. (The discount rate ρ vanishes because $o(e^{-\rho dt}) = o(dt) < o(dQ)$.)

Since B draws immediately at t^+ , we know that p and V are given by Proposition 2: $V(Q) = (y - c(Q))/\rho$ and $p = c'(Q)/\rho$. Substituting and rearranging, we have

that

$$c(Q^+ + d\tilde{Q}) - c(Q + d\tilde{Q}) \leq c'(Q^+ + d\tilde{Q})(Q^+ - Q). \quad (49)$$

There are two cases, corresponding to $Q^+ \leq Q$:

- If $Q^+ > Q$, then equation (49) says

$$\frac{c(Q^+ + d\tilde{Q}) - c(Q + d\tilde{Q})}{Q^+ - Q} \leq c'(Q^+ + d\tilde{Q}). \quad (50)$$

That contradicts the concavity of c , as desired.

- If $Q^+ < Q$, then equation (49) says

$$\frac{c(Q + d\tilde{Q}) - c(Q^+ + d\tilde{Q})}{Q - Q^+} \geq c'(Q^+ + d\tilde{Q}). \quad (51)$$

That also contradicts the concavity of c , as desired.

□

The analysis so far says that it suffices to compare drawing immediately to a sequence of jumps $\{Q_{t_n}\}_n$. But, by the Markov assumption, at each Q_{t_n} , B either jumps immediately or stays there forever. (Staying at Q_{t_n} for an interval and then jumping later would violate the Markov assumption, which implies that at Q_{t_n} you either jump or not.) Immediate jumps are telescoping—they cancel out. So it is sufficient to compare drawing to jumping to an absorbing state. That is what we do in the next two lemmata.

Lemma C.3. *There is no absorbing state with $Q^+ > Q^{\tilde{c}}$.*

Proof. Suppose (in anticipation of a contradiction) that Q^+ is an absorbing state.

B must prefer not to draw at Q^+ :

$$\frac{y - c(Q^+)}{\rho} > \tilde{p}d\tilde{Q} + V(Q^+ + d\tilde{Q}) \quad (52)$$

or, substituting $V(Q)$ from Proposition 2,

$$\frac{y - c(Q^+)}{\rho} \geq \tilde{p}d\tilde{Q} + \frac{y - c(Q^+ + d\tilde{Q})}{\rho} \quad (53)$$

or

$$\frac{1}{\rho} \frac{c(Q^+ + d\tilde{Q}) - c(Q^+)}{d\tilde{Q}} \geq \tilde{p}. \quad (54)$$

Combined with condition (19) that implies that

$$\frac{1}{\rho} \frac{c(Q^+ + d\tilde{Q}) - c(Q^+)}{d\tilde{Q}} \geq \frac{1}{\rho} \frac{c(Q^{\tilde{c}} + d\tilde{Q}) - c(Q^{\tilde{c}})}{d\tilde{Q}}, \quad (55)$$

which, given $Q^+ > Q^{\tilde{c}}$, contradicts the concavity of c .

□

Lemma C.4. *Suppose $p^{\tilde{c}} < \gamma(Q^-)/\rho$ for $Q^- < Q^{\tilde{c}}$. B prefers to jump to $Q^{\tilde{c}}$ than to any possible absorbing state $Q^- < Q^{\tilde{c}}$.*

Proof. We want to show that

$$p^{\tilde{c}}(Q^{\tilde{c}} - Q) + \tilde{V}(Q^{\tilde{c}}) \geq p^-(Q^- - Q) + \frac{y - c(Q^-)}{\rho} \quad (56)$$

or, using condition (18),

$$p^{\tilde{c}}(Q^{\tilde{c}} - Q) + \tilde{p}d\tilde{Q} + V(Q^{\tilde{c}} + d\tilde{Q}) \geq p^-(Q^- - Q) + \frac{y - c(Q^-)}{\rho} \quad (57)$$

or, substituting in for V from Proposition 2,

$$p^{\tilde{c}}(Q^{\tilde{c}} - Q) + \tilde{p}d\tilde{Q} + \frac{y - c(Q^{\tilde{c}} + d\tilde{Q})}{\rho} \geq p^-(Q^- - Q) + \frac{y - c(Q^-)}{\rho} \quad (58)$$

or, adding and subtracting $c(Q^{\tilde{c}})/\rho$ and then rearranging,

$$\left\{ \tilde{p}d\tilde{Q} \right\} + \left[p^-(Q - Q^-) - p^{\tilde{c}}(Q - Q^{\tilde{c}}) \right] \geq \left\{ \frac{1}{\rho} (c(Q^{\tilde{c}} + d\tilde{Q}) - c(Q^{\tilde{c}})) \right\} + \left[\frac{1}{\rho} (c(Q^{\tilde{c}}) - c(Q^-)) \right]. \quad (59)$$

Now it suffices to compare just the terms in curly/square brackets:

- Curly brackets:

$$\tilde{p} \geq \frac{1}{\rho} \frac{c(Q^{\tilde{c}} + d\tilde{Q}) - c(Q^{\tilde{c}})}{d\tilde{Q}}. \quad (60)$$

That is satisfied by condition (19) of the Proposition.

- Square brackets:

$$p^-(Q - Q^-) - p^{\tilde{c}}(Q - Q^{\tilde{c}}) \geq \frac{1}{\rho} (c(Q^{\tilde{c}}) - c(Q^-)). \quad (61)$$

Now given that $Q - Q^{\tilde{c}} > 0$, if $p^- > p^{\tilde{c}}$, it suffices to show the inequality with $p^{\tilde{c}}$ replaced with p^- :

$$p^-(Q - Q^-) - p^-(Q - Q^{\tilde{c}}) \geq \frac{1}{\rho} (c(Q^{\tilde{c}}) - c(Q^-)) \quad (62)$$

or

$$p^-(Q^{\tilde{c}} - Q^-) \geq \frac{1}{\rho} (c(Q^{\tilde{c}}) - c(Q^-)) \quad (63)$$

or, given $Q^{\tilde{c}} > Q^-$,

$$p^- \geq \frac{1}{\rho} \frac{c(Q^{\tilde{c}}) - c(Q^-)}{Q^{\tilde{c}} - Q^-}. \quad (64)$$

Recall that Q^- is a supposed absorbing state, so $p^- = \gamma(Q^-)/\rho$. The inequality becomes

$$\gamma(Q^-) \geq \frac{c(Q^{\bar{c}}) - c(Q^-)}{Q^{\bar{c}} - Q^-}, \quad (65)$$

a sufficient condition for which is

$$\gamma(Q^-) \geq c'(Q^-), \quad (66)$$

which is satisfied for Q^- below Q^e (cf. Proposition 1 and condition (18)).

□

Lemma C.5. *Suppose that Q^0 and Q^1 are both absorbing states with $Q^0 < Q^1 < Q^e$. B prefers to jump from $Q > Q^1$ to Q^1 than to Q^0 .*

Proof. We want to show that

$$p^1(Q^1 - Q) + \frac{y - c(Q^1)}{\rho} > p^0(Q^0 - Q) + \frac{y - c(Q^0)}{\rho} \quad (67)$$

or, rearranging,

$$\frac{1}{\rho} \left(c(Q^1) - c(Q^0) \right) < p^0(Q - Q^0) - p^1(Q - Q^1) \quad (68)$$

$$= p^0(Q - Q^0) - \underbrace{p^0(Q - Q^1) + p^0(Q - Q^1)}_{=0} - p^1(Q - Q^1) \quad (69)$$

$$= p^0(Q^1 - Q^0) + (p^0 - p^1)(Q - Q^1). \quad (70)$$

That holds whenever

$$\frac{1}{\rho} \frac{c(Q^1) - c(Q^0)}{Q^1 - Q^0} < p^0 + (p^0 - p^1) \frac{Q - Q^1}{Q^1 - Q^0}. \quad (71)$$

Given c is concave, $p^0 = \gamma(Q^0)/\rho$, and the last term is positive ($p^0 > p^1$, $Q > Q^1$, and $Q^1 > Q^0$), a sufficient condition is that

$$c'(Q^0) < \gamma(Q^0), \tag{72}$$

which is satisfied since $Q^0 < Q^e$.

□

Lemma C.6. *For $Q^- < Q^{\tilde{c}}$, $p^- > p^{\tilde{c}}$.*

Proof. Suppose (in anticipation of a contradiction) that $p^- < p^{\tilde{c}}$. Given the arguments above, in particular the “corollary” argument following Lemma C.2), we know three things:

1. Q_t must jump down from $Q^{\tilde{c}}$ (otherwise the price would be higher);
2. Q_t must jump to an absorbing state immediately, say Q^b (otherwise the policy would either be smooth, violating Lemma C.1, or would stay at a point for some time before jumping, violating the Markov property);
3. the absorbing state must be below Q^- , $Q^b < Q^-$ (otherwise the price at Q^b would be above that at Q^- , violating the hypothesis that $p^{\tilde{c}} > p^-$).

B’s policy must, of course, be optimal: At $Q^{\tilde{c}}$, he must prefer to jump to Q^b than to Q^- . But that contradicts Lemma C.5. We conclude that $p^- > p^{\tilde{c}}$, as desired.

□

Lemma C.7. *For $Q < Q^{\tilde{c}}$, B prefers not to draw.*

Proof. Here we show that when $Q < Q^{\tilde{c}}$, B prefers to jump to $Q^{\tilde{c}}$ than to draw the credit line (which implies only that drawing the line is not optimal, not that

jumping to $Q^{\tilde{c}}$ is):

$$\tilde{p}d\tilde{Q} + V(Q + d\tilde{Q}) < p(Q^{\tilde{c}})(Q^{\tilde{c}} - Q) + \tilde{V}(Q^{\tilde{c}}) \quad (73)$$

or, substituting in for V and \tilde{V} and rearranging,

$$\tilde{p}d\tilde{Q} < \frac{\gamma(Q^{\tilde{c}})}{\rho}(Q^{\tilde{c}} - Q) + \frac{c(Q + d\tilde{Q}) - c(Q^{\tilde{c}})}{\rho}. \quad (74)$$

The result follows from observing that the RHS above exceeds the LHS for all $Q > Q^{\tilde{c}}$. To see that, observe first that it holds with equality at $Q = Q^{\tilde{c}}$ and that the RHS does not depend on Q whereas the LHS is decreasing in it; that follows from differentiation:

$$\frac{c'(Q + d\tilde{Q})}{\rho} - \frac{\gamma(Q^{\tilde{c}})}{\rho} < \frac{c'(d\tilde{Q})}{\rho} - \frac{\gamma(Q^e)}{\rho} < 0, \quad (75)$$

given $d\tilde{Q}$ is large and the definition of Q^e in equation (8).

□

C.5 Proof of Lemma 1

Existence (of $\tilde{p} > 0$) follows immediately from c being increasing: Just set

$$\tilde{p} := \frac{1}{\rho} \frac{c(Q + d\tilde{Q}) - c(Q)}{d\tilde{Q}}. \quad (76)$$

Uniqueness follows immediately from c concave: $(c(Q + d\tilde{Q}) - c(Q))/d\tilde{Q}$ is strictly monotonic, hence intersects the constant $\rho\tilde{p}$ at most once.

C.6 Proof of Proposition 4

We prove the proposition in three steps. First, we show that with a credit line satisfying the conditions of the Proposition in place, then $Q^{\tilde{c}}$ is an absorbing state (Lemma C.8). Second, we use this result to fully characterize the value function when such a credit line is in place (Lemma C.9). Finally, we verify that such a credit line always exists (Lemma C.10).

Lemma C.8. *If a credit line $(\tilde{p}, d\tilde{Q})$ satisfying the conditions of the Proposition is in place, then $Q^{\tilde{c}}$ is an absorbing state.*

Proof. We show first that there is no issuance, i.e. no lender would buy at a price at which B would sell:

- Proposition 4 and the tie-breaking rule in Assumption 3 imply that lenders know B will draw for $Q > Q^{\tilde{c}}$. Now the result follows from Assumption 2, inequalities (23) and (25), and the assumption that c' is bounded above zero as follows: $\Gamma(Q_t) = \int_0^\infty e^{-\rho s} \gamma(Q_{t+s}) ds \leq \int_0^\infty e^{-\rho s} \gamma(Q_t) e^{k_0 + k_1 s} ds = \gamma(Q_t) e^{k_0} / (\rho - k_1)$ with $\gamma(Q) \rightarrow 0$ as $Q \rightarrow \infty$. Thus, as condition (26) in the statement is satisfied, lenders do not lend at any price B would accept.

We now show that there are no buy backs, i.e. no lender would sell at a price B would buy:

- As $Q^{\tilde{c}} \leq Q^e$ (the exclusive allocation debt level that maximizes gains from trade) it follows from Assumption 1.2 that $\gamma(Q^{\tilde{c}}) > c'(Q^{\tilde{c}})$ and thus there are also no gains from offering debt buybacks.

We conclude that $Q^{\tilde{c}}$ must be an absorbing state—there is no issuance and no buyback—and so $\tilde{V}(Q^{\tilde{c}}) = \frac{y - c(Q^{\tilde{c}})}{\rho}$.

□

Lemma C.9. *If $Q^{\tilde{c}}$ is an absorbing state, then the value function is as stated in the Proposition.*

Proof. The result follows from what we have established already:

- For $Q > Q^{\tilde{c}}$, B draw the credit line by Proposition 3 and Assumption 3.
- For $Q < Q^{\tilde{c}}$, B jump to absorbing state $Q^{\tilde{c}}$ by Proposition 3 (see Lemma C.7 in the proof).
- $Q^{\tilde{c}}$ is an absorbing state by Lemma C.8 above.

□

Lemma C.10. *Given \tilde{V} above, a credit line satisfying the properties in the Proposition exists.*

Proof. In light of Lemma C.8 and Lemma C.9 above, the result follows immediately from Lemma 1. □

C.7 Proof of Proposition 5

First, suppose (in anticipation of a contradiction) that B takes up any bundle inducing a different outcome at date 0 in equilibrium. By Proposition 4, another lender can offer a bundle that implements Q^e and, by Proposition 1, it breaks even at p^e . By the definition of the full-commitment outcome, B is strictly better off. Thus, since B accepts at most one contract at each date, there is $\epsilon > 0$ such that the lender can offer $(p^e - \epsilon, Q^e)$ and both B and the lender (that was previously getting zero) are strictly better off. That is a profitable deviation and therefore a contradiction to the proposed equilibrium. Second, B is indifferent between drawing or not at all time, so he does not draw. Finally, it is easy to verify that $Q^e < \bar{Q}$ as Q^e is finite and given by the solution to $\gamma(Q^e) = c'(Q^e) - \gamma'(Q^e)Q^e < c'(\infty)$. □

C.8 Proof of Lemma 2

The result follows from equation (31) and (15), which implies that $V' = -c'/\rho$ for $dt \rightarrow 0$. □

C.9 Proof of Proposition 6

Immediate from inequalities (32) and (33). □

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